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| **Problem 2.29** | A piece of n-type silicon is doped with $10^{17}$ $cm^{-3}$ shallow donors. Calculate the density of electrons per unit energy at $kT/2$ above the conduction band edge. $T$ = 300 K. Calculate the electron energy for which the density of electrons per unit energy has a maximum. What is the corresponding probability of occupancy at that maximum? |

**Solution**

The density of electrons per unit energy at a given energy equals:

$$n(E) = g_c(E)\, f(E)$$

where

$$g_c(E) = \frac{8\sqrt{2}p\ m^{3/2}}{h^3}\sqrt{E - E_c} = 1.05 \text{ cm}^{-3}\text{J}^{-1}$$

and

$$f(E) = \frac{1}{1 + \exp\frac{E - E_F}{kT}} = 9.14 \text{ x } 10^{-4}$$

The position of the Fermi energy is calculated from the doping density:

$$E_F - E_c = kT \ln\frac{n}{N_c} = kT \ln\frac{N_d}{N_c} = -\ 168 \text{ meV}$$

This last equation is only valid if the semiconductor is non-degenerate, which is a justifiable assumption since the electron density is much smaller than the effective density of states. The Fermi function then becomes:

$$f(E) = \frac{1}{1 + \exp\frac{E - E_F}{kT}} \cong \exp\frac{E_F - E}{kT}$$

And the density of electrons per unit energy can then be further simplified to:

$$n(E) = g_c(E)\, f(E) \cong \frac{8\sqrt{2}p\ m^{3/2}}{h^3}\sqrt{E - E_c}\ \exp\frac{E_F - E}{kT}$$

The maximum is obtained by setting the derivative equal to zero:

$$\frac{dn(E)}{dE} = 0$$

This result in:

$$0 = \frac{1}{2}\frac{1}{\sqrt{E - E_c}}\exp\frac{E_F - E}{kT} - \frac{1}{kT}\sqrt{E - E_c}\ \exp\frac{E_F - E}{kT}$$

Which can be solved to yield:

$$E = E_{\text{max}} = E_c + kT/2$$

The corresponding probability of occupancy equals the value of the Fermi function calculated above.