
Problem 1.9 Prove that the probability of occupying an energy level below the Fermi energy equals the probability that an energy level above the Fermi energy and equally far away from the Fermi energy is not occupied.

Solution The probability that an energy level with energy ΔE **below** the Fermi energy E_F is occupied can be rewritten as:

$$\begin{aligned} f(E_F - \Delta E) &= \frac{1}{1 + \exp \frac{E_F - \Delta E - E_F}{kT}} = \frac{\exp \frac{\Delta E}{kT}}{\exp \frac{\Delta E}{kT} + 1} \\ &= 1 - \frac{1}{\exp \frac{\Delta E}{kT} + 1} = 1 - \frac{1}{1 + \exp \frac{E_F + \Delta E - E_F}{kT}} = 1 - f(E_F + \Delta E) \end{aligned}$$

so that it also equals the probability that an energy level with energy ΔE **above** the Fermi energy, E_F , is **not** occupied.
