Problem 1.8 Derive the electric field of a proton with charge q as a function of the distance from the proton using Gauss's law. Integrate the electric field to find the potential $\phi(r)$:

$$\phi(r) = \frac{q}{4\pi\varepsilon_0 r}$$

Treat the proton as a point charge and assume the potential to be zero far away from the proton.

Solution Using a sphere with radius, r, around the charged proton as a surface where the electric field, E, is constant, one can apply Gauss's law:

$$E(r)4\pi r^2 = \frac{q}{\varepsilon_0}$$

so that

$$E(r) = \frac{q}{4\pi r^2 \varepsilon_0}$$

The potential is obtained by integrating this electric field from to Resulting in:

$$\phi(r) - \phi(\infty) = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{q}{4\pi\varepsilon_0 r}$$

where the potential at infinity was set to zero.