Problem 1.5 Derive equations (1.2.9) and (1.2.10). Calculate the total energy as the sum of the kinetic and potential energy.

Solution The derivation starts by setting the centrifugal force equal to the electrostatic force:

$$m_0 \frac{v^2}{r} = \frac{p^2}{m_0 r} = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

where the velocity, v, is expressed as a function of the momentum, p.

The momentum in turn is calculated as a function of the deBroglie wavelength and the wavelength must be an integer fraction of the length of the circular orbit

$$\frac{p^2}{mr} = \frac{h^2}{mr\lambda^2} = \frac{h^2 n^2}{mr4\pi^2 r^2} = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

The corresponding radius equals the Bohr radius, a_0:

$$a_0 = \frac{\varepsilon_0 h^2 n^2}{\pi m_0 q^2}$$

The corresponding energies are obtained by adding the kinetic and potential energy:

$$E_n = \frac{p^2}{2m_0} - \frac{q^2}{4\pi\varepsilon_0 a_0} = -\frac{m_0 q^4}{8\varepsilon_0^2 h^2 n^2} \text{, with } n = 1, 2, \ldots$$

Note that the potential energy equals the potential of a proton multiplied with the electron charge, $-q$.