Modern Physics - Exam 2 Name:______________________________

1. The negative muon is a subatomic particle with the same charge as the electron but a mass that is about 207 times greater. A muon can be captured by a proton to form a "muonic hydrogen" atom with energy and radius given by the Bohr model, except that the electron's mass must be replaced by the muon's mass. Derive the energy and radius of the first Bohr orbit in a muonic hydrogen atom. What is the wavelength range of the muonic Blamer series (transitions ending in the n=2 level)?

2. Derive an expression for the electron's speed in the nth Bohr orbit in a hydrogen atom. Prove that the orbit with the highest speed is the n=1 orbit. Compare this with the speed of light, and comment on the validity of ignoring relativity (as we did) in discussing the hydrogen atom.

3. Using the relativistic relation between E and p ($E^2 = p^2c^2 + m^2c^4$), show that electrons and photons with the same energy E have different wavelengths. (Note: The deBroglie relation $p = h/\lambda$ is relativistically correct.) When do their wavelengths approach equality?

4. A beam of particles of energy E, incident upon a (semi-infinite) potential step of $V_0 = \frac{5}{4}E$, is described by the wavefunction $\psi_{inc}(x) = e^{ikx}$. Determine completely the reflected wave and the wave inside the step by enforcing the required continuity conditions to obtain their (possibly complex) amplitudes. Verify by explicit calculation that the ratio of reflected probability density to the incident probability density is unity.

5. If a particle in a stationary state is bound, the expectation value of its momentum must be zero. In words, why? Then, prove it! Starting from the general expression for an expectation value, integrate by parts, then argue that the result is identically zero. (Be careful that your argument is somehow based on the particle being bound; a free particle certainly may have a nonzero momentum. Note: $\psi(x)$ may always be chosen to be real in a stationary state.)